A STUDY OF HEAT PROOFING MATERIALS UNDER TRANSIENT CONDITIONS

B. M. Pankratov, O. M. Alifanov,
A. A. Ivanov, and A. D. Markin

UDC 536.491:536.421.1

Theoretical and experimental methods of studying heat proofing materials are evaluated. Some characteristics of transient breakdown of fusable and cokable coatings are examined.

Some Problems Concerning the Transient Breakdown of Fusable Materials. Heat- and mass-transfer processes occuring during the travel of a body through the atmosphere of a planet will always be transient in nature, because the flow conditions vary. In calculating the transiency of heatup and breakdown one usually assumes the body to be at every instant of time in a stream with constant parameters and one does not consider that, besides the parameters which characterize the outer stream ρ_∞ and V_∞, there are also their time derivatives $d\rho_\infty/dt$ and dV_∞/dt. In other words, the state of flow at every instant of time must be described not only by the density the velocity but also by their time derivatives.

For this reason, there is a need for developing an appropriate theory and then a method by which the effect of transiency on the heatup and the breakdown of a body in a stream can be accounted for.

This problem has been considered in the case of heat proofing materials destructable by fusion and evaporation (vitreous materials).

The system of equations describing the breakdown of such a material at the critical point can be represented as follows [3]:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}-\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\rho F_x,$$

$$\frac{\partial p}{\partial y}=\rho F_y,$$

$$\frac{\partial(\rho ux)}{\partial x}+\frac{\partial(\rho vx)}{\partial y}=0,$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2.$$

For a more complete and accurate description of this process, it is necessary to add also a system of transient-state equations for the gaseous boundary layer and to solve them simultaneously. In view of the complexity involved here, these systems of equations were analyzed separately and the transiency of the ambient stream are taken into account in the change to dimensionless quantities:

$$\eta=\sqrt{\frac{2\rho\beta(t)}{\mu_*}}\,y,\quad \frac{\partial f}{\partial\eta}=\frac{u(y,\,t)}{u_e(t)},$$

$$\tau=\int_0^t\beta(t)\,dt,\ \theta(\tau,\,\eta)=\frac{T(t,\,y)-\bar{T}}{T_e(t)}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 24, No. 1, pp. 75-83, January, 1973. Original article submitted May 22, 1972.

in the vicinity of the critical point. When changing to these dimensionless quantities, one must consider that the velocity gradient near the critical point is a function of time. After the necessary transformations, we have

$$-\frac{1}{2}\cdot\frac{\partial^2 f}{\partial\tau\partial\eta}-\frac{\partial}{\partial\eta}\left(\frac{\mu}{\mu_*}\cdot\frac{\partial^2 f}{\partial\eta^2}\right)+f\,\frac{\partial^2 f}{\partial\eta^2}-\frac{1}{2}\left[\left(\frac{\partial f}{\partial\eta}\right)^2-\frac{\rho_e}{\rho}\right]=\frac{a}{2}\left(\frac{\eta}{2}\cdot\frac{\partial^2 f}{\partial\eta^2}+\frac{\partial f}{\partial\eta}-\frac{1}{A}\right),$$

$$-\frac{1}{2}\cdot\frac{\partial\theta}{\partial\tau}+\frac{1}{\sigma_*}\cdot\frac{\partial}{\partial\eta}\left(\bar{\lambda}\,\frac{\partial\theta}{\partial\eta}\right)+f\,\frac{\partial\theta}{\partial\eta}+a\left(-\frac{\eta}{4}\cdot\frac{\partial\theta}{\partial\eta}+\Phi_1\theta+\Phi_2+\Phi_3\right)=0,$$

where

$$A=\sqrt{\frac{\bar{\gamma}_e-1}{\bar{\gamma}_e}},$$

coefficient $a=\dot{\beta}/\beta^2$ accounts for the variation of the stream parameters (u_e, p_e, T_e) with time, coefficients Φ_1, Φ_2, Φ_3 are defined by the steady-state values of the stream parameters, $\sigma = C_p\mu_*/\lambda_*$, $\bar{\lambda}=\lambda/\lambda_*$. Before analyzing the effect of "outer" transiency on the vehicle trajectory, one should first answer the question concerning the ranges of parameter values and trajectory characteristics within which this effect on the breakdown of vitreous materials is significant, i.e., to first solve the problem qualitatively. Since this would require a great amount of parametric calculations, the procedure will be somewhat modified instead.

We consider the steady-state heatup and breakdown of a heat proofing coat on the surface of a body in a gaseous stream, described by the parameters (u_e, p_e, T_e) as well as by their time derivatives. This situation is physically unrealizable, as an analysis of the procedure for solving transient-state equations (by the finite-differences method with the use of the implicit scheme) indicates. The following system of equations is solved:

$$\frac{\partial}{\partial\eta}\left(\frac{\mu}{\mu_*}\cdot\frac{\partial^2 f}{\partial\eta^2}\right)+f\,\frac{\partial^2 f}{\partial\eta^2}-\frac{1}{2}\left[\left(\frac{\partial f}{\partial\eta}\right)^2-\frac{\rho_e}{\rho}\right]=\frac{a}{2}\left(\frac{\eta}{2}\cdot\frac{\partial^2 f}{\partial\eta^2}+\frac{\partial f}{\partial\eta}-\frac{1}{A}\right),$$

$$\frac{1}{\sigma_*}\cdot\frac{\partial}{\partial\eta}\left(\bar{\lambda}\,\frac{\partial\theta}{\partial\eta}\right)+f\,\frac{\partial\theta}{\partial\eta}+a\left(-\frac{\eta}{4}\cdot\frac{\partial\theta}{\partial\eta}+\Phi_1\theta+\Phi_2+\Phi_3\right)=0.$$

Calculations for a wide range of parameter values and trajectory characteristics show that the material wear and the temperature profile are affected most significantly by mass forces and by the transiency of the stagnation temperature. The results of calculations made for one point on the trajectory are shown in Fig. 1. Taking into account the transiency of the stagnation temperature yields lower temperatures, according to the graph, inasmuch as trajectories are considered here for which $\partial T_e/\partial t < 0$. The opposite will occur when $\partial T_e/\partial t > 0$, namely the vehicle will accelerate. It is to be noted that the maximum deviation in the temperature profiles $\Delta\bar{T} = (T_{\alpha=0}-T_{\alpha=1})$ corresponds to a zero deviation in the thermal flux profiles (θ').

As the flight altitude becomes lower, the difference $\Delta\bar{T} = (T_{\alpha=0}-T_{\alpha=1})/(T_{w_{\alpha=0}}-T_\infty)$ decreases, until the surface temperature reaches its maximum, and then increases again (Fig. 2, where $\gamma = a/\sigma_* f_\infty^2$), i.e., the effect of stagnation temperature transiency is most pronounced along the trajectory segments with a relatively low surface temperature. As a consequence, the effect of trajectory characteristics on the magnitude of $\Delta\bar{T}_{max}$ and thus on wear in the liquid phase becomes obvious. Inasmuch as the thermophysical properties (viscosity and thermal conductivity) of a vitreous material such as fused quartz have not yet been determined exactly, we will now analyze how the temperature characteristics of thermal conductivity and viscosity affect the deviations in the temperature profiles. Calculations show that they are affected most strongly by $\mu(T)$ and less strongly by $\lambda(T)$.

Thus, available test data on the thermophysical properties of this material are not sufficient for definitively evaluating the effect of these properties on deviations in the temperature profile and on wear in the liquid phase.

Cokable Heat Proofing Materials. The breakdown of intricate composite materials under heavy heat loads is accompanied by the formation of a pyrolysis zone inside, by the filtration of gaseous products to the hot surface, and by heat transfer between solid and gas. The effect of these factors on the heatup and the wear of heat proofing materials can be analyzed by means of a mathematical model of the process with the following assumptions:

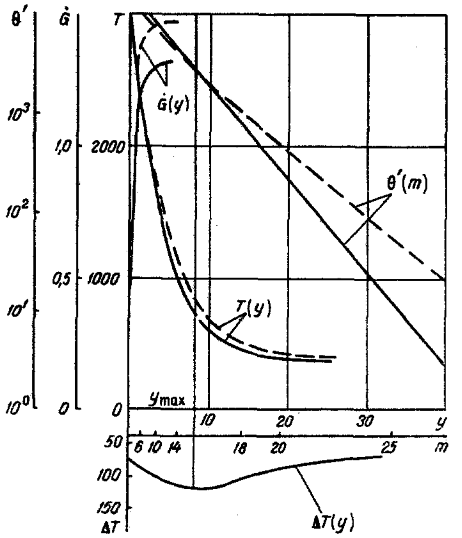

Fig. 1

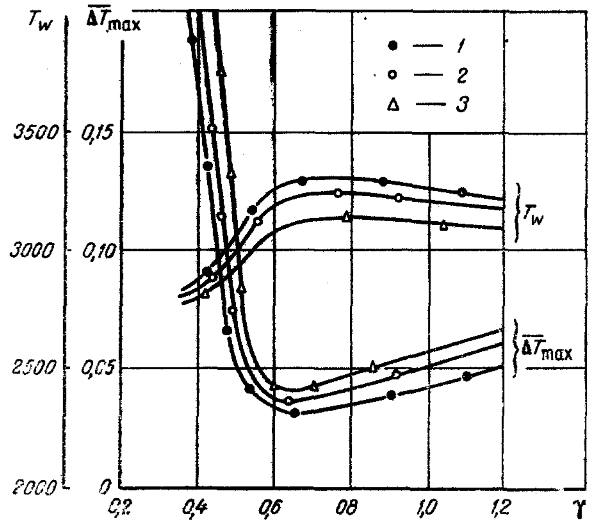

Fig. 2

Fig. 1. Effect of the transiency of the gas stream on the temperature profile (V_f = 7000 m/sec, θ_f = 30°, $\sigma = 10^{-4}$ m^2/N, h = 33 km, R_B = 0.4 m): solid lines) α = 1; dashed lines) α = 0.

Fig. 2. Variation of the maximum deviations in a temperature profile along a trajectory (V_f = 7000 m/sec, $\sigma = 10^{-4}$ m^2/N, R_B = 0.4 m): 1) θ_f = 45°; 2) 30; 3) 15.

1. Chemical reactions within the coke zone occur only in the gaseous state, the gases flowing through the pores at a constant velocity. Both the Lewis number and the Prandtl number are taken equal to unity.
2. The heat load in this analysis corresponds to a one-dimensional (in space) problem.
3. Physicochemical changes within the zone where the bonding material undergoes pyrolysis are described by the equations of nonisothermal kinetics:

for n = 1

$$\frac{\partial \rho_{res}}{\partial t} = -B\rho_{res}\exp[-E/RT] - \frac{E}{RT^2}\rho_{res}\ln\left(\frac{\rho_{res}}{\rho_0}\right)\frac{\partial T}{\partial t},$$

for n ≠ 1

$$\frac{\partial \rho_{res}}{\partial t} = -B\left(\frac{\rho_{res}}{\rho_0}\right)^n \rho_0\exp[-E/RT] - \frac{E\rho_0}{RT^2(1-n)}\left[\left(\frac{\rho_{res}}{\rho_0}\right)^n - \frac{\rho_{res}}{\rho_0}\right]\frac{\partial T}{\partial t},$$

$$T_{phys} = \varphi\left(\frac{\partial T}{\partial t}\right).$$

4. The boundaries of the pyrolytic zone are defined by the initial temperature of physicochemical changes and by the tar concentration, the latter being equal to zero at the outer edge. Under these assumptions concerning the physicochemical processes which occur in the breaking down of material and concerning the boundaries of the various zones, the transient temperature field of a heat proofing coat is described by the following system of heat-and-mass-transfer equations:

for the zone of undecomposed material

$$\rho C_p\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right);$$

for the zone of decomposed bonding material

$$\rho C_p\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + W,$$

where

$$W = -I_0 \frac{\partial \rho_{res}}{\partial t}.$$

and for the zone of residual coke

$$\rho_C C_{pC} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_C \frac{\partial T}{\partial x}\right) - \frac{\alpha_0}{C_{pG}} K_1 (H_C - H),$$

$$\rho_G \frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\lambda_G}{C_{pG}} \cdot \frac{\partial T_G}{\partial x}\right) - (\rho u)_G \frac{\partial H}{\partial x} - \frac{\alpha_0}{C_{pG}} K_2 (H - H_C),$$

where

$$(\rho u)_G = \int_{x_1}^{x_2} (1 - cc) \frac{\partial \rho}{\partial t} dx.$$

Subscripts C and G refer to the solid phase and the gaseous phase, respectively. The initial condition is $T(x, 0) = T_0$. The boundary conditions are

$$q_1(t)|_{x=0} + \lambda \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0; \; q_2(t)\Big|_{x=X(t)} + \lambda_C \left.\frac{\partial T}{\partial x}\right|_{x=X(t)} = 0,$$

$$T_G|_{x=X_C(t)} = T_{x=X_C(t)}; \; q_2(t)|_{x=X(t)} + \lambda_G \left.\frac{\partial T_G}{\partial x}\right|_{x=X(t)} = 0.$$

Thus, a numerical solution to this system of partial differential equations yields the transient temperature field of a material undergoing physicochemical changes, with gas filtration and heat transfer in the coke layer taken into account.

Experimental Determination of Transient Boundary Conditions. It must be noted that the experiment alone is very significant in a study of heat transfer phenomena. In view of the necessity to fundamentally modify the procedural and the design aspects of the experiment, which includes data processing and final evaluation, "transient-state" experiments must differ qualitatively from experiments which have so far been designed for steady-state heat-transfer studies. This requires that a theoretical basis be developed for transient-state thermal experiments, with particular emphasis on methods of establishing the boundary conditions. Such an effort is justified by the fact that the correct tracking or the proper control of thermal flux and surface temperature variations in a body with time constitute the basis of experimental studies concerning the transient heat transfer.

In most cases, the only way to establish the thermal boundary conditions for transient heat transfer is to solve reverse problems of heat conduction. These problems belong in the category of noncorrective problems, because in the course of their formulation one violates the stability condition for the sought solution. Nevertheless, under certain conditions, the problem can be solved by direct methods. We propose here a numerical method of solving the nonlinear reverse problem of one-dimensional heat conduction in a homogeneous plate.

The solution to this problem is constructed by the implicit scheme of finite-differences for approximating the quasilinear equation of heat conduction:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\bar{a}} \cdot \frac{\partial T}{\partial t} - \frac{\lambda'}{\lambda}\left(\frac{\partial T}{\partial x}\right)^2,$$

where

$$\bar{a} = \frac{\lambda}{\rho C_p}, \; \lambda' \equiv \frac{d\lambda}{dT}, \; \lambda = \lambda(T), \; C_p = C_p(T), \; \rho = \rho(T).$$

For a six-point scheme [1] we have

$$-\frac{\bar{a}_{i,n} \Delta t}{(\Delta x)^2} T_{i-1,n+1} + \left[1 + \frac{2\bar{a}_{i,n}\Delta t}{(\Delta x)^2}\right] T_{i,n+1} - \frac{\bar{a}_{i,n}\Delta t}{(\Delta x)^2} T_{i+1,n+1} = T_{i,n} + \frac{\bar{a}_{i,n} \Delta t}{(\Delta x)^2}\left(\frac{\lambda'_{i,n}}{4\lambda_{i,n}}\right)(T_{i+1,n} - T_{i-1,n})^2, \quad (1)$$

where

$$\Delta x = \frac{b}{k};\quad \Delta t = \frac{t_m}{m};\ i=1,2,\ \dots,\ k-1;\ n=0,1,\ \dots,\ m-1;$$

$$\bar{a}_{i,n} = \bar{a}(T_{i,n});\ \lambda_{i,n} = \lambda(T_{i,n}).$$

Assuming a thermally insulated wall, for simplicity, we can write the boundary conditions as follows:

$$q_{n+1} - \lambda_{0,n}\frac{T_{0,n+1}-T_{1,n+1}}{\Delta x} = (C_{p0,n})\,\frac{\Delta x}{2}\cdot\frac{T_{0,n+1}-T_{0,n}}{\Delta t} \tag{2}$$

at x = 0 and

$$-\lambda_{k,n}\frac{T_{k,n+1}-T_{k-1,n+1}}{\Delta x} = (C_{pk,n})\,\frac{\Delta x}{2}\cdot\frac{T_{k,n+1}-T_{k,n}}{\Delta t} \tag{3}$$

at x = b.

The initial condition is

$$T_{i,0} = f(x_i),\ i = 0,\ 1,\ \dots,\ k.$$

With the aid of relations (1)-(3), one can write a system of linear algebraic equations for the unknown thermal flux q_{n+1} and the unknown temperatures $T_{i,n+1}$ ($i = 0, \dots, \varkappa-1,\ \varkappa+1, \dots, k$ with the known temperature $T_\varkappa$ at a given point $i = \varkappa$ inside the body).

Without showing all the mathematical transformations, we have

$$Az = b,$$

$$A = \begin{vmatrix}
d_{0,n} & 2l_{0,n} & & & & & & & & & -1\\
l_{1,n} & d_{1,n} & l_{1,n} & & & 0 & & & & & 0\\
 & l_{2,n} & d_{2,n} & l_{2,n} & & & & & & & 0\\
 & & \cdot & & & & & & & & \cdot\\
 & & & \cdot & & & & & & & \cdot\\
 & & & \cdot & & & & & & & \cdot\\
 & & & l_{\varkappa-2,n} & d_{\varkappa-2,n} & l_{\varkappa-2,n} & & & & & 0\\
 & & & & l_{\varkappa-1,n} & d_{\varkappa-1,n} & & & & & 0\\
 & & & & & l_{\varkappa,n} & l_{\varkappa,n} & & & & 0\\
 & & & & & & d_{\varkappa+1,n} & l_{\varkappa+1,n} & & & 0\\
 & 0 & & & & & l_{\varkappa+2,n} & d_{\varkappa+2,n} & l_{\varkappa+2,n} & & 0\\
 & & & & & & & & & & \cdot\\
 & & & & & & & & & & \cdot\\
 & & & & & & & & 2l_{k,n} & d_{k,n} & 0
\end{vmatrix},$$

$$z = \begin{bmatrix} T_{0,n+1}\\ T_{1,n+1}\\ T_{2,n+1}\\ \cdot\\ \cdot\\ \cdot\\ T_{\varkappa-2,n+1}\\ T_{\varkappa-1,n+1}\\ T_{\varkappa+1,n+1}\\ T_{\varkappa+2,n+1}\\ T_{\varkappa+3,n+1}\\ \cdot\\ \cdot\\ f_{0,n} \end{bmatrix},\qquad
b = \begin{bmatrix} 0\\ b_{1,n}\\ b_{2,n}\\ \cdot\\ \cdot\\ \cdot\\ b_{\varkappa-2,n}\\ b_{\varkappa-1,n} - l_{\varkappa-1,n}T_{\varkappa,n+1}\\ b_{\varkappa,n} - d_{\varkappa,n}T_{\varkappa,n+1}\\ b_{\varkappa+1,n} - l_{\varkappa+1,n}T_{\varkappa,n+1}\\ b_{\varkappa+2,n}\\ \cdot\\ \cdot\\ T_{k,n} \end{bmatrix},$$

where

$$d_{i,n} = 1 + 2\,\frac{\bar{a}_{i,n}\Delta t}{(\Delta x)^2},\quad l_{i,n} = -\frac{\bar{a}_{i,n}\Delta t}{(\Delta x)^2},$$

$$b_{i,n} = T_{i,n} + \frac{\bar{a}_{i,n}\Delta t}{(\Delta x)^2}\left(\frac{\lambda'_{i,n}}{4\lambda_{i,n}}\right)(T_{i+1,n} - T_{i-1,n})^2.$$

In this way, from the known temperatures $T_{i,n}$ and $T_{\varkappa,n+1}$ we find $T_{i,n+1}$ and

$$q_{n+1} = \frac{\lambda_{0,n}\Delta x}{2\bar{a}_{0,n}\Delta t}\,[f_{0,n} - T_{0,n}].$$

This scheme for solving the reverse problem of heat conduction was successfully applied to a number of cases. In order to obtain stable results at sufficiently small computation steps along the time coordinate, it is necessary to first smooth out the input data $T_\delta(t)$. When the error (δ) in $T_\delta(t)$ is large and fluctuates, then the input function is shaped by the regularization method [2].

The proposed method of establishing the boundary conditions is suitable for solving many problems related to flight testing of various vehicles or related to the experimental simulation of transient modes in heat proofing systems.

NOTATION

ρ	is the density of the material;
C_p	is the specific heat of the material;
λ	is the thermal conductivity of the material;
μ	is the dynamic viscosity of the material;
x, y	are the coordinates;
u, v	are the velocity components along x- and y-axis, respectively;
t	is the time;
τ	is the dimensionless time;
u_e, p_e, T_e, ρ_e	are the velocity, pressure, temperature, and density of the gas at the outer edge of the boundary layer;
$\bar{\gamma}_e$	is the mean ratio of specific heats of the gas at the outer edge of the boundary layer;
η	is the dimensionless space coordinate;
f	is the dimensionless velocity;
θ	is the dimensionless temperature;
$\dot{G}$, f_∞	are the actual and dimensionless rate of material breakdown;
$\bar{\lambda}$	is the relative thermal conductivity;
F_x, F_y	are the components of mass force along the x-axis and along the y-axis;
T	is the temperature;
T_W, T_∞	are the temperature at the outside and the inside surface of the material;
ΔT, $\Delta\bar{T}$	are the absolute and relative deviation in a temperature profile;
θ'	is the dimensionless thermal flux;
β, $\dot{\beta}$	are the velocity gradient and its time derivative;
γ, a	are the transiency coefficients;
$\bar{T}$, μ_*, A, λ_*, σ_*, K_1, K_2, B	are the constants;
V_f	is the velocity of vehicle entry into the atmosphere;
θ_f	is the angle of vehicle entry into the atmosphere;
R_B	is the bluntness radius of vehicle;
σ	is the ballistic coefficient;
h	is the flight altitude;
$\alpha = 0$	is the transiency of gas stream parameter disregarded;
$\alpha = 1$	is the transiency of gas stream parameter accounted for;
Φ_1, Φ_2, Φ_3	are the coefficients in the energy equation;
ρ_0, ρ_{res}	are the initial and any instantaneous density of bonding resin;
E	is the activation energy;
R	is the universal gas constant;
n	is the reaction number;
α_0	is the heat-transfer coefficient;
$T_{phys.}$	is the initial temperature of physicochemical change;
W	is the power of heat source;
I_0	is the total caloric value of material;
H	is the enthalpy;
CC	is the coke number;
$X(t)$, $X_C(t)$	are the laws according to which the boundaries of the coke zone shift;

q	is the thermal flux absorbed by the body;
q_1, q_2	are the laws governing the heat transfer at the inside and at the outside surface of the material, respectively;
b	is the plate thickness;
T_δ	is the "perturbed" inlet temperature in the reverse problem of heat conduction;
t_m	is the time interval for computing the right-hand boundary value;
k	is the number of layers in a plate along the space coordinate;
m	is the number of steps along the time coordinate;
p	is the pressure.

LITERATURE CITED

1. M. Lotkin, J. Math. and Phys., No. 2 (1958).
2. V. A. Morozov, Zh. Vychisl. Matem. i Fiz., No. 4 (1967).
3. W. G. Dorrens, Supersonic Flow of Viscous Gases [Russian translation], Mir (1966).